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Brussel Sprouts - Game

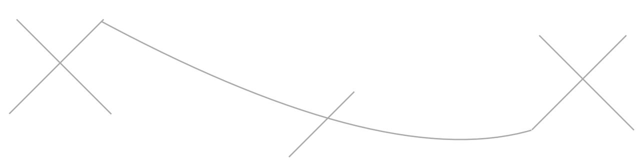
[This lesson plan seeks to take an existing game, and get students to move from the playing of the game to the mathematics which lies behind it.]

Description of the Game

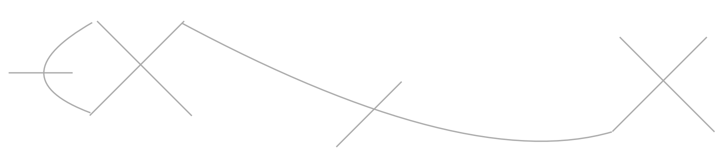
Start with two unconnected crosses:

On your turn, you have to join one free end of a cross to another free end of a cross (You can join it to a free end of the same cross as well). The only rule is that the join cannot intersect with a join which already exists. Once you have create an edge, draw a short line through the center of the edge to create two new free ends, which can now be used as starting points for joints

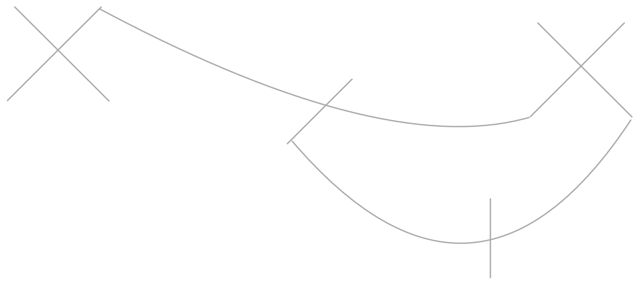
Example of the first player's first move:



Example of the second player's first move:

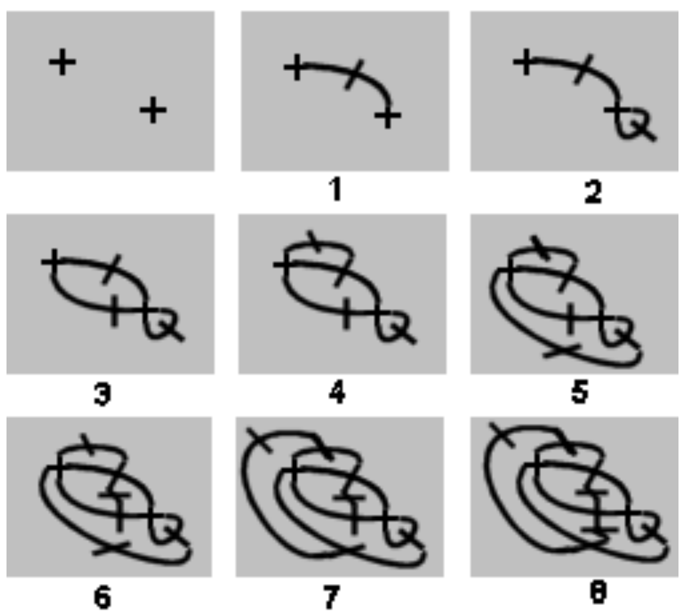


Another example of the second player's first move:



The last person to make a legitimate move wins.

Example of a complete game of Brussel Sprouts (from Wikipedia)



Task 1

With a partner try out the game 20 times, taking turns being first and second. Note down each time whether the first player wins or the second. Do you see a pattern?

	Player 1	Player 2	Winner
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

State the pattern:

Task 2

Now, rather than two crosses, try three crosses to start off with. Try the same things again 10 times. Was there a pattern this time? Was the pattern the same?

	Player 1	Player 2	Winner
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

State the pattern:

Try and guess what the pattern will be for 4, 5 and 6 crosses:

Task 3

Try out the game starting with 4, 5 and 6 crosses (five times each). Does the pattern you predicted in Task 2 actually work?

4	Player 1	Player 2	Winner
1			
2			
3			
4			
5			

5	Player 1	Player 2	Winner
1			
2			
3			
4			
5			

6	Player 1	Player 2	Winner
1			
2			
3			
4			
5			

Does the pattern hold?

What is your prediction for 2378973249 crosses? What about 32894798332 crosses? Can you state the general pattern for n crosses?

Task 4

Go back to task 1 (with 2 crosses). This time, rather than just seeing who wins, count the number of moves as well.

	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

Is there a pattern in the number of moves? If this pattern is always true, does it explain the pattern you found in Task 1?

Is there anything you can say about the number of moves it would take for 3 crosses?

Task 5

Count the number of moves for one cross, and for three crosses.

1	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

3	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

Is there a pattern in how the number of moves changes as the number of crosses increases?

Predict the number of moves with 4 crosses and 5 crosses.

Task 6

Check the predictions you made about the number of moves for four and five crosses

4	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

5	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

Now, what is your prediction for 6 crosses? What about for 15 crosses?

Task 7

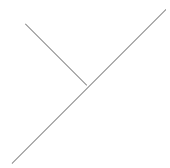
Lets put down all the information we have gotten so far:

Number of Crosses	Winner (P1 or P2)	Number of Moves
1		
2		
3		
4		
5		

Can you now state a pattern for *n* crosses such that you can easily calculate (with a calculator, of course!) what you predict the number of moves will be for 7834983274928 crosses or 32847983221 crosses?

Task 8

Now, rather than crosses, try starting with Ys.



1 Y	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

2 Y	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

3 Y	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

State the patterns you find, both in the winner and in the number of moves:

Task 9

Check to see if the patterns continue for 4 and 5 Ys:

4 Y	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

5 Y	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

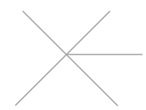
Now, fill in the following, like you did for the crosses:

Number of Ys	Winner (P1 or P2)	Number of Moves
1		
2		
3		
4		
5		

Can you state the general formula you predict for *n* Ys?

Task 10

Now, try starting with a shape with five end points.



1-5	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

2-5	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

3-5	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

State the patterns you find, both in the winner and in the number of moves:

Task 11

Check to see if the patterns continue for 4 and 5 shapes with 5 endpoints:

4-5	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

5-5	Player 1	Player 2	Winner	Number of Moves
1				
2				
3				
4				
5				

Now, fill in the following, like you did for the Ys and the crosses:

Number of 5s	Winner (P1 or P2)	Number of Moves
1		
2		
3		
4		
5		

Can you state the general formula you predict for n 5s?

Task 12

Fill in the following:

Number of Endpoints per shape	Formula for Number of Moves for n shapes
3	
4	
5	

Can you predict what the formula will be for shapes with 6 end-points, and 20 end points?

State the formula for n shapes each with m endpoints:

Continuing the Game

Part 1

So, far we have only dealt with homogenous configurations. By that, I mean only crosses or only Ys and so on. We have not tried configurations like starting with a Y and a cross together, or a shape with five endpoints, a shape with six endpoints, and a shape with 14 endpoints.

Are there patterns in these mixed configurations? Try finding them.

Part 2

This part is much harder. Try coming up with mathematical proofs for the patterns you found above. What you have done so far approaches scientific proofs - proofs involving data and samples. But how do we know that a game of Brussel Sprouts in the Andromeda Galaxy with two Wookies playing, will behave in the same way. We think it will, right? You need to come up with an argument for why these patterns work always.