

Classification of Shapes

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Learning Outcomes:

1. Evaluating Classificatory Systems
2. Choosing between classificatory systems

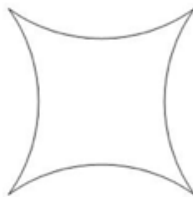
T: What is a square?

S: A square is a shape with equal sides.

T: So, an equilateral triangle is a square?

S: No, a square is a shape with 4 sides all of which are equal.

T: So, this is a square:

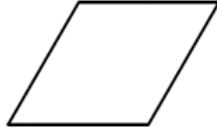


S: No. A square is a shape with 4 straight line sides, all of which are equal.

T: So, is this a square:

S: No, the angles have to be 90 degrees.

T: Okay, for now let us assume we know what 90 degrees means. Is this a square:



S: No. It has to be closed.

T: Okay, so let us recap the properties which distinguish a square from other shapes:

1. It is closed
2. It has 4 sides
3. Its sides are straight lines
4. All angles are perpendicular
5. All sides are equal

Anything which has all these properties is a square and anything missing even one of these is not. So, this counts as a definition of a square.

Let us move to another question: Is a square a rectangle?

S: No. A square has all sides equal while a rectangle has only opposite sides equal.

T: Since a square has all sides equal, it also has opposite sides equal. So, why is a square not a rectangle?

S: Well, when we look at a square table, we don't call it a rectangular table, and when somebody tells us that a table is rectangular, we don't consider that it might be square. However, I remember reading in a textbook that a square is a rectangle.

T: For now, let us ignore textbooks and you try to answer the question as to whether you think a square should be a rectangle or not.

S: For the reasons I gave earlier, clearly a square should not be a rectangle.

T: Let us write down the definition of a rectangle:

1. It is closed
2. It has 4 sides
3. Its sides are straight lines
4. All angles are perpendicular
5. Opposite sides are equal
6. Adjacent sides are not equal

Notice, that if we keep 6, a square is not a rectangle. However, if we get rid of 6, a square is a rectangle.

We are choosing between the following classificatory systems:

Figure 1: Competing Classificatory Systems



S: As we said before, in order to describe things around us A is probably better.

T: I agree. However, let us look at the classification in another way – if we accept B, everything true about a rectangle is automatically inherited by squares. For instance, you may have learnt that the diagonals of a rectangle are equal. Well that is also true about squares. If we choose classification A, we have to prove this separately for squares. In classification B, if we prove it for rectangles, it is automatically inherited by the square. So, even though A is better for descriptive purposes, B is better for ‘academic’ purposes. Also, notice that the definition of square is a lot shorter in B – a square is a rectangle with adjacent sides equal.

If this did not convince you, let me try something else. You probably know what parallelograms and quadrilaterals are. I will provide two classifications for quadrilaterals, parallelograms, squares and rectangles. I want you to come up with their definitions like we did in the case of squares and rectangles.

Figure 2: Competing Classificatory System - A

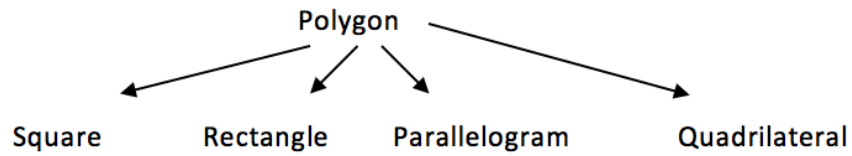
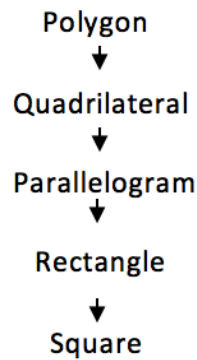


Figure 3: Competing Classificatory System - B



Let us assume all of these are types of Polygons – Polygons are closed shapes with straight lines as sides. Start with B.

S: A Quadrilateral is a 4-sided polygon.

A Parallelogram is a Quadrilateral with opposite sides parallel.

A Rectangle is a Parallelogram with all right angles.

A Square is a Rectangle with adjacent sides equal.

T: That was pretty easy. Now try A. Start with a square. Then you will have to define a rectangle in such a way that it is not a square, a parallelogram such that it is not a rectangle nor a square, and a quadrilateral such that it is none of a square, rectangle or parallelogram.

S: (This will take a while and a lot of help to get to)

A Square is a 4-sided polygon with all sides equal and all right angles.

A Rectangle is a 4-sided polygon with all right angles, opposite sides equal, and adjacent sides not equal.

A Parallelogram is a 4-sided polygon with opposite sides parallel and no right angle.

A Quadrilateral is a 4-sided polygon with no pair of parallel sides.

T: I think all of your definitions work except for the last one. Look at the following. It has one pair of parallel sides, but is not a parallelogram, rectangle or square:



S: Okay, a Quadrilateral is a 4-sided polygon with at most one pair of parallel sides.

T: Notice that the A was a lot harder than B, and the definitions were longer. So, we have two reasons for choosing B over A:

- a. Inheritance of properties
- b. Simpler definitions

Note: This can be generalized to areas outside of mathematics. For instance, consider the choice between the Linnaean Classificatory system of living organisms vs Aristotle's classification. Linnaeus classed humans, great apes, and so on while Aristotle chose to separate humans from all other living creatures. If our goal is to make predictions about humans and other living things, the reasons for choosing Linnaeus over Aristotle are similar to the ones above.