



# Murder

## Probabilistic Thinking Lesson Plan

### Key Idea

We deal with different types and levels of uncertainty each day of our lives. Many times resolving this uncertainty requires putting together various factors, each of which have uncertainty associated with them. This is clearly evident in the legal and medical professions, where the diagnosis/judgement is based on factors including testimony, experience, expertise, and established knowledge. Being able to think probabilistically in these situations is crucial in order to come to sound judgements/diagnoses.

### Learning Outcomes

An intuitive understanding of:

1. Probability as a degree of certainty in a belief
2. Probabilistic logic (and, or, & conditional)
3. Bayesian updating

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## Learning Task

The following is a dialogue between a mother and her son. The mother is putting forward a hypothetical scenario involving a murder trial, and her son is attempting to come to a conclusion on the guilt of the defendant based on the information the mother is giving. Once new information emerges, the mother pushes the son to update his degree of certainty.

**Mother:** Did you hear about this person who was arrested for murder? They are appearing in court for their trial. What is your degree of belief that they will be found guilty?

**Son:** Shouldn't we be open to hear the case and not take a judgement?

**Mother:** By 'open' do you mean 50-50 or more towards innocent or more towards guilty.

**Son:** 50-50 seems arbitrary. It would mean that about half the time I would expect the person to be found guilty just as in a coin toss I would expect to get heads around half the time

**Mother:** Can we base our starting point on something less arbitrary?

**Son:** How about basing it on the percentage of people who are guilty of murder?

**Mother:** Well, that would be less than 0.0001% of people. Are you saying that before any more information about the defendant, you are almost certain of them being found innocent?

**Son:** No, that seems too low. The difference here is that the person who has been arrested is not an arbitrary person. That person has been arrested.

**Mother:** So, how about restricting ourselves to people who have been arrested and find the percentage of those who have subsequently been found to be guilty?

**Son:** That would probably be significantly higher and it is not as arbitrary as starting with 50-50. If we started with a 0.0001% belief, that would imply that the police are completely incompetent and just arrest random, unconnected people for a given murder. This new rate is probably much closer to the truth.

**Mother:** Lets assume that 30% of people who have been arrested have been found guilty. Now, let me give you some more information about the defendant. The defendant is a woman. Also, there are far fewer women murderers than men murderers. Does that change your degree of belief?

**Son:** Shouldn't that decrease my degree of confidence in the defendant being found guilty? Just as the defendant being arrested increased my confidence in their guilt, this should decrease it. So, my degree of confidence in her being found guilty should be less than 30%.

**Mother:** We need to be a little careful here. It is true that in the population women are much less likely to be found guilty as murderers. However, our 30% was not based on the general population, but on the arrested population.

**Son:** So, we need to look at the percentage of women who are guilty of murder out of those who have been arrested.

**Mother:** Yes!

**Son:** Hmm.. So, the information you gave about the rate of women being murderers was not important here. I suspect that the percentage of guilty women out of those who have been arrested is probably about the same as the percentage of guilty men out of those who have been arrested.

**Mother:** Possibly. However, let me paint you another plausible narrative. Assume the police know that women are far less likely to be murderers than men. So, in order for them to arrest a woman, they would require far more evidence than for them to arrest a man. Hence, women who have been arrested may be far more likely to be guilty than their male counterparts. So, the information I gave you could possibly result in you increasing your degree of confidence in your defendant's guilt. However, that is just plausible speculation. Lets assume your conclusion was right and the percentage of guilty women out of those who have been arrested is similar to the percentage of guilty men out of those who have been arrested.

**Son:** So, nothing changes with this new information. Whats the next bit of information?

**Mother:** Lets say that the victim was killed by poison and that out of every 100 murders committed by poison, 45 are by women. Does that change anything?

**Son:** Would it make it more likely that the murder was a man?

**Mother:** Once again, this requires some care. We need to know what the percentage of women murderers are. Lets say 20% of murders are committed by women. In that case if out of 100 murders with poison, 45 are committed by women, this information should actually increase our belief that the murder was committed by a woman.

**Son:** This probability stuff is really hard!

**Mother:** You're getting the hang of it. Does this change your degree of belief in your defendant's guilt?

**Son:** It does increase my belief that the murderer was a women, though after all the other things we have gone through, I am very unsure about that conclusion. Does that mean that it increases my belief that the murderer was this particular woman? If it does, by how much?

**Mother:** Well, that depends on who the other possible murderers are. If all the other possible murderers are women, it should not have any effect. If there are some male suspects, it could either increase or decrease your belief in the guilt of the defendant. If all the other suspects are male, your degree of belief in the defendant's guilt will increase. So, we need to explore who the other suspects are.

**Son:** Who are they?

**Mother:** The other suspects the police have questioned were two men and one woman. They have also ruled out suicide.

**Son:** I'm sure the percentage of non-arrested suspects being guilty is significantly less than arrested suspects being guilty. So, if we are saying the arrested woman has a 30% chance of

having committed the murder (without the information about the poison), each of these other people must have less than a 10% chance.

**Mother:** Lets say there is a 5% chance that somebody suspected of murder by the police but not arrested is guilty of murder.

**Son:** There seems to be a problem here. If we add up the chance of the four suspects having done the murder, we get a total of 45%. However, we are assuming there is certainty that the person was actually murdered. So, shouldn't it sum up to 100%?

**Mother:** It should sum up to 100% if all possible suspects have been accounted for. Are we missing out on suspects?

**Son:** Maybe the police didn't ever suspect the person who actually did the murder. Maybe the person is somebody apart from the four suspects we have so far.

**Mother:** If we accept what we have done so far, there is a 55% chance that the suspect is somebody else.

**Son:** To see how the poison information effects our degree of certainty, we need to have an idea of how many of this 55% are male and how many are female.

**Mother:** Should we say 50-50?

**Son:** Rather than that, should we use the information you gave about the percentage of murderers who are women?

**Mother:** Since the person we are looking for is a murderer, we can do that. So, 20% of the 55% are women and 80% of the 55% are men.

**Son:** So, lets note down the suspects with their chance of having committed the murder:

<b>Suspect</b>	<b>Gender</b>	<b>Chance</b>
Arrested Suspect	Female	30%
Suspect A	Male	5%
Suspect B	Male	5%
Suspect C	Female	5%
Other Female	Female	11%
Other Male	Male	44%

**Mother:** So, even though our main suspect is female, there is only a 46% percent chance that the murder was committed by a woman. However, that is more than the 45% of poisonings committed by women. So, our degree of belief that the murder was one of the female suspects actually goes down with that information, albeit slightly. It will go down so slightly though, that it will not make much impact.

**Son:** Can you give more details about the case?

**Mother:** Sure. Lets say that the victim was the defendant's uncle. The uncle was killed in his house, where he lived alone. There was no sign of any forced entry into the house.

**Son:** That would make it likely that the murderer was somebody he knew. In fact, the method of murder, poison, also points to somebody he knew rather than a stranger. That is assuming the poison was administered just before his death rather than much before.

**Mother:** The poison was found in a wine glass, poured from a bottle of wine which had just been opened. The wine in the bottle had no trace of poison.

**Son:** So, now we need to return to the list of suspects and figure out which of them was known to the uncle and which were not.

**Mother:** Out of the three other suspects, one of the males and females was known to the uncle. The third one, a man, was a door to door salesman of religious books who was seen knocking on the uncle's door.

**Son:** Was the uncle religious or interested in religion?

**Mother:** Not from what we have heard about him from others.

**Son:** I guess it is still possible that the uncle invited the salesman in for a glass of wine, but it seems less likely. So, our degree of certainty in the guilt of our defendant and the two suspects who know the uncle goes up, while the degree of certainty in the guilt of the salesman goes down.

**Mother:** What do we do about the 'Others'?

**Son:** Well, out of other possible suspects we had before, the uncle may have known some of them. But, how many?

**Mother:** So, our previous assumption of 55% for others was based on people who were in the area when the uncle was murdered - those with the opportunity to murder the uncle. Think about the people you see everyday who have the opportunity to murder you. How many of them do you know?

**Son:** Well, I go to school every day. I know 10% of the people in my school - it is a huge school.

**Mother:** The uncle was retired and spent most of his time alone at home. Most of the people who he talked to were people he knew. However, before we had the information about the lack of breaking in and so on, there is a significant chance that the murder was committed by a stranger who had broken in.

**Son:** So, say our 55% at that stage was made up of 20% of people he knew and 80% of people he didn't know, and let that be equally divided amongst the women and men:

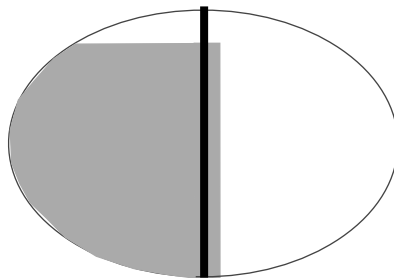
Suspect	Gender	Knew the Victim?	Chance
Arrested Suspect	Female	Yes	30%

Suspect A	Male	Yes	5%
Suspect B	Male	No	5%
Suspect C	Female	Yes	5%
Other Female he Knew	Female	Yes	2.2%
Other Male he Knew	Male	Yes	8.8%
Other Female he Didn't know	Female	No	8.8%
Other Male he Didn't know	Male	No	35.2%

So, at that stage, 51% of suspects were known to the uncle and 49% were not. With the new information about the form of the murder, it makes it much more likely that the murder was committed by somebody he knew.

**Mother:** Lets say that in cases with poison and without a break-in, there is a 90% chance that the murder will be committed by a person known to the uncle. So, there is a 90% chance that the 51% of suspects known to the uncle committed the murder, and a 10% chance that the suspects not known committed the murder.

**Son:** So, if we look at the diagram below, let the left of the dividing line represent suspects the uncle knows and the right hand side of the line represent the suspects the uncle didn't know. The filled in gray portion represents the chance of the them being murderers. There is a 90% chance that the left hand side has a murderer and a 10% chance that the right hand side does.



We are only concerned with those who have murdered, so we restrict ourselves to the gray area. This results in just over a 90% chance that the murderer was somebody the uncle knew and just under a 10% chance that the murderer was somebody the uncle didn't know.

**Mother:** So, we need to revise the table. The total of the people he knew should add up to 90, while the total of the people he didn't know should add up to 10. We should make the adjustments in proportion:



<b>Suspect</b>	<b>Gender</b>	<b>Knew the Victim?</b>	<b>Chance</b>	<b>Updated Chance</b>
Arrested Suspect	Female	Yes	30%	54%
Suspect A	Male	Yes	5%	9%
Suspect B	Male	No	5%	1%
Suspect C	Female	Yes	5%	9%
Other Female he Knew	Female	Yes	2.2%	4%
Other Male he Knew	Male	Yes	8.8%	14%
Other Female he Didn't know	Female	No	8.8%	2%
Other Male he Didn't know	Male	No	35.2%	7%

**Son:** So, for the first time, our degree of certainty in the guilt of the defendant is over 50%.

**Mother:** So, if the trial ended at this stage, and you had to make a decision on the guilt of the defendant, would you judge her to be guilty and send her to jail? Remember, there is no way you will ever have complete certainty.

**Son:** It is still too close for me to make a decision. I would not be happy to make a decision so critical on just a coin toss. The phrase I have heard used here is: 'Beyond reasonable doubt.' At what point is it beyond reasonable doubt?

**Mother:** That is a decision we have to make which cannot be dictated by data or information. It can be guided by precedent and by questions about the chances of freed murderers re-offending. However, at the end, you as the judge will have to make a decision on whether you are okay with 90% certainty or 75% or even 99.9% before you convict.

**Son:** Even 99.9% will result in 1 in 1000 convicted people being innocent. That seems very unfair.

**Mother:** Look at it the other way: if we require 99.9% certainty, then many more guilty will get away since there may not be sufficient evidence to convict them. The formulation used in the British justice system formulated by jurist William Blackstone is:

*“It is better that ten guilty persons escape than that one innocent suffer”*

**Son:** So, the question we need to ask is: How many guilty get away at various levels of certainty?

**Mother:** Lets leave that for another time. We will also get back to the murder case at some point.